$\int \cos^4 x \, dx = \int [\cos^2 x]^2 \, dx$

$\int \left(\frac{1 + \cos 2x}{2}\right)^2 dx$

$\frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x \, dx$

$\frac{1}{4}\int 1 + 2\cos 2x + \frac{1 + \cos 4x}{2} \, dx$

$\frac{1}{4}\int \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x \, dx$

$\frac{3x}{8} + \left(\frac{1}{4}\right)\left(\frac{2\sin 2x}{2}\right) + \frac{1}{8}\frac{\sin 4x}{4} + C$

$\frac{3x}{8} + \cancel{2}\frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$

~~sec~~ $\int \tan^n x \, \underline{\sec^m x} \, dx$ $\quad$ m is even, $u = \tan x$

$\int \tan^4 x \sec^2 x \, dx$ $\quad$ $u = \tan x$

$du = \sec^2 x \, dx$

$\int u^4 \, du$

$\frac{u^5}{5} + C$

$\boxed{\frac{\tan^5 x}{5} + C}$

$\int \tan^3 x \sec^4 x \, dx$ $\quad$ $u = \tan x$

$du = \sec^2 x \, dx$

$\int \tan^3 x \sec^2 x \, \underline{\sec^2 x \, dx}$

$\int \tan^3 x (\tan^2 x + 1) \sec^2 x \, dx$

$\int u^3 (u^2 + 1) \, du$

$\int u^5 + u^3 \, du$

$\frac{u^6}{6} + \frac{u^4}{4} + C$

$\frac{\tan^6 x}{6} + \frac{\tan^4 x}{4} + C$

$\int \tan^n x \sec^m x\, dx$

n is odd

$\int \tan^3 x \sec^3 x\, dx$

$u = \sec x$

$du = \sec x \tan x\, dx$

$\int \tan^2 x \sec^2 x \underline{\sec x \tan x\, dx}$

$\int (\sec^2 x - 1)(\sec^2 x) \sec x \tan x\, dx$

$\int (u^2 - 1) u^2\, du$

$\int u^4 - u^2\, du$

$\frac{u^5}{5} - \frac{u^3}{3} + C$

$\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$

$\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x\, dx$

$= -\int \frac{1}{u} du = -\ln|u|$

$= \cancel{\ln|\tan}$

$= -\ln|\cos x| + C$

$= \ln|\sec x| + C$

$\int \tan^3 x\, dx = \int \tan x \tan^2 x\, dx$

$= \int \tan x (\sec^2 x - 1)\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \int \tan x \sec^2 x\, dx - \int \tan x\, dx$

$= \int u\, du$

$= \frac{\tan^2 x}{2} - \ln|\sec x| + C$

$\int \sin^m x \cos^n x \, dx$

m is odd
or n is odd

$$\int \cos^3 x \, dx$$

$$\int \cos^2 x \cos x \, dx$$

$$\int (1-\sin^2 x) \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int 1 - u^2 \, du$$

$$u - \frac{u^3}{3} + C$$

$$\boxed{\sin x - \frac{\sin^3 x}{3} + C}$$

$$\int \sin^5 x \cos^4 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$\int \sin^4 x \cos^4 x \sin x \, dx$$

$$\int (1-\cos^2 x)^2 \cos^4 x \sin x \, dx$$

$$\int (1 - 2\cos^2 x + \cos^4 x) \cos^4 x \sin x \, dx$$

$$\int (\cos^4 x - 2\cos^6 x + \cos^8 x) \sin x \, dx$$

$$-\int u^4 - 2u^6 + u^8 \, du$$

$$-\frac{u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9} + C$$

$$-\frac{\cos^5 x}{5} + \frac{2\cos^7 x}{7} - \frac{\cos^9 x}{9} + C$$

$$\int_0^{\pi} \sin^2 x \, dx$$

$u = 2x$

$du = 2 \, dx$

$\frac{du}{2} = dx$

$$\int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$$

$$\frac{1}{2} \int_0^{\pi} 1 - \cos 2x \, dx$$

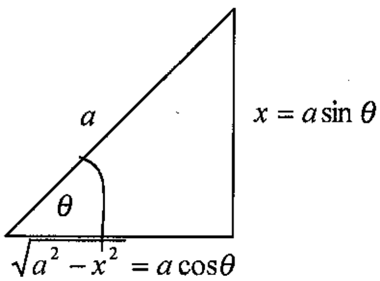
$$\frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$

$$\frac{1}{2} \left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right) \right] = \boxed{\frac{\pi}{2}}$$

Trigonometric Substitution

For $\sqrt{a^2 - x^2}$:   Use $x = a\sin\theta$

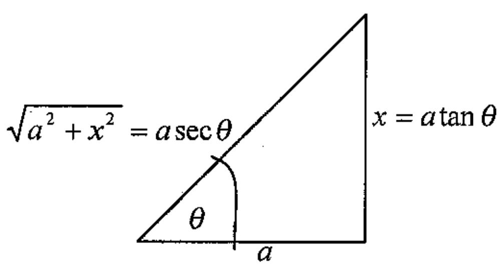
Then $\sqrt{a^2 - x^2} = a\cos\theta$



$$\int x^3\sqrt{4 - x^2}\, dx$$

For $\sqrt{a^2 + x^2}$:   Use $x = a\tan\theta$

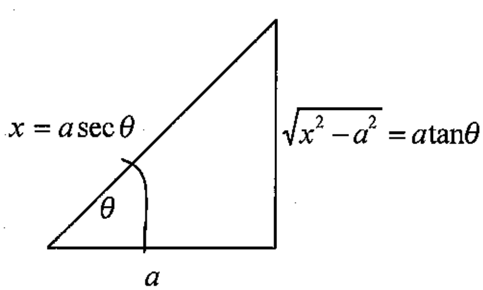
Then $\sqrt{a^2 + x^2} = a\sec\theta$



$$\int \frac{1}{\sqrt{x^2 + 16}}\, dx$$

For $\sqrt{x^2 - a^2}$:   Use $x = a\sec\theta$

Then $\sqrt{x^2 - a^2} = a\tan\theta$



$$\int_0^3 \sqrt{x^2 + 6x}\, dx$$

$\int \sec\theta \, d\theta$ and $\int \sec^3\theta \, d\theta$

$\int \sec\theta \, d\theta$

$= \int \sec\theta \left[\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \right] d\theta$

$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\tan\theta + \sec\theta} \, d\theta$ ~~Note: Numerator is~~ derivative of denominator

$u = \tan\theta + \sec\theta$

$= \int \frac{1}{u} \, du$ $du = (\sec^2\theta + \sec\theta\tan\theta) \, d\theta$

$= \ln|u| + C = \boxed{\ln|\sec\theta + \tan\theta| + C = \int \sec\theta \, d\theta}$

$\int \sec^3\theta \, d\theta$ $u = \sec\theta$ $dv = \sec^2\theta \, d\theta$

$= \int \sec^2\theta \sec\theta \, d\theta$ $du = \sec\theta\tan\theta \, d\theta$ $v = \tan\theta$

$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta \, d\theta$

$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1) \, d\theta$

$= \sec\theta\tan\theta - \int \sec^3\theta \, d\theta + \int \sec\theta \, d\theta$

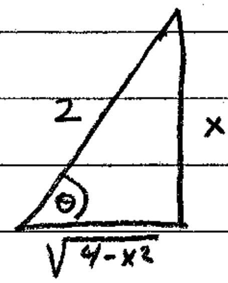
$\int \sec^3\theta \, d\theta = \sec\theta\tan\theta - \int \sec^3\theta \, d\theta + \ln|\sec\theta + \tan\theta| + C$

$2\int \sec^3\theta \, d\theta = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C$

$\boxed{\int \sec^3\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C}$

#1

$$\int x^3\sqrt{4-x^2}\,dx$$



$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$$\int 8\sin^3\theta\, 4\cos^2\theta\, d\theta$$

$$32\int \sin\theta\,(1-\cos^2\theta)\cos^2\theta\, d\theta$$

$$32\int \sin\theta\left(\cos^2\theta - \cos^4\theta\right) d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$-32\int u^2 - u^4\, du$$

$$-\frac{32}{3}u^3 + \frac{32}{5}u^5 + C$$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

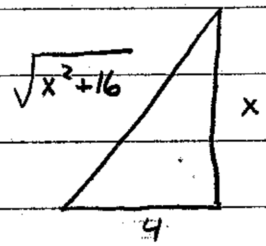
$$-\frac{32}{3}\cos^3\theta + \frac{32}{5}\cos^5\theta + C$$

$$\left(-\frac{32}{3}\right)\left(\frac{1}{8}\right)\left(\sqrt{4-x^2}\right)^3 + \left(\frac{32}{5}\right)\left(\frac{1}{32}\right)\left(\sqrt{4-x^3}\right)^5 + C$$

$$\boxed{\frac{1}{5}\sqrt{(4-x^2)^5} - \frac{4}{3}\sqrt{(4-x^3)^3} + C}$$

#2

$$\int \frac{dx}{\sqrt{x^2+16}}$$



$x = \tan\theta$

$dx = 4\sec^2\theta \, d\theta$

$\sqrt{x^2+16} = 4\sec\theta$

$$= \int \frac{4\sec^2\theta \, d\theta}{4\sec\theta}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$ (see previous work)

$$= \ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C$$

$$= \ln\left|\frac{x+\sqrt{x^2+16}}{4}\right| + C$$

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#3

$$\int_0^3 \sqrt{x^2+6x}\; dx = \int_0^3 \sqrt{x^2+6x+9-9}\; dx$$

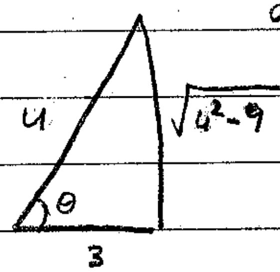
$$= \int_0^3 \sqrt{(x+3)^2-9}\; dx$$

$u = x+3$

$du = dx$

x	u
0	3
3	6

$$= \int_3^6 \sqrt{u^2-9}\; du$$



$u = 3\sec\theta$

$du = 3\sec\theta\tan\theta\, d\theta$

$\sqrt{u^2-9} = 3\tan\theta$

$$= 9\int_0^{\pi/3} \tan\theta(\sec\theta\tan\theta)\, d\theta$$

$$= 9\int_0^{\pi/3} \sec\theta\tan^2\theta\, d\theta$$

$\sec\theta = \frac{u}{3}$

u	$u/3$	θ
3	1	0
6	2	$\pi/3$

$$= 9\int_0^{\pi/3} \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= 9\int_0^{\pi/3} \sec^3\theta - \sec\theta\; d\theta$$

$$= 9\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta|\right]_0^{\pi/3}$$

$$= \left(\frac{9}{2}\sec\theta\tan\theta - \frac{9}{2}\ln|\sec\theta+\tan\theta|\right)\Big|_0^{\pi/3}$$

$$= \left[\frac{9}{2}\sec\frac{\pi}{3}\tan\frac{\pi}{3} - \frac{9}{2}\ln\left|\sec\frac{\pi}{3}+\tan\frac{\pi}{3}\right|\right] - \left[\frac{9}{2}\sec 0\tan 0 - \frac{9}{2}\ln|\sec 0+\tan 0|\right.$$

$$= \left(\frac{9}{2}\right)(2)(\sqrt{3}) - \left(\frac{9}{2}\right)\ln|2+\sqrt{3}| - \left(\frac{9}{2}\right)(1)(0) + \frac{9}{2}\ln|1+0|$$

$$= 9\sqrt{3} - \frac{9}{2}\left(\ln|2+\sqrt{3}| + \ln|1|\right)$$

$$= \boxed{9\sqrt{3} - \frac{9}{2}\ln(2+\sqrt{3})}$$